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Effect of slip on Herschel- Bulkley Fluid Flow Through An Artery With Stenosis and Post Stenotic Dilatation

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Abstract: Slip effect on steady Herschel-Bulkley fluid flow through a tube with stenosis and post stenosis dilation have been investigated. It is assumed that the stenosis is mild. The analytical expressions are obtained for resistance to the flow, velocity, and pressure drop and wall shear stress. The different parameters effects on the flow rate, shear stress and stream lines are discussed and presented graphically.

Keywords : Herschel-Bulkley fluid, Slip velocity,Stenoses, Dilatation.

1. INTRODUCTION

Cardiovascular diseases are one of the major health problems one such is atherosclerosis. The malfunction of blood flow due to stenosis causes a severe circulatory disorder. The entire cardiovascular system is affected due to the presence of stenosis and because of this it is one of the interesting areas of research .

Shukla *et. al.*[15] studied the non-Newtonian behaviour of blood flow in the stenosed artery, whereas Mishra et al. [8] analysed by considering stenosed artery of a uniform cross-section. Sreenadh *et al.* [17] developed a mathematical model to study the multiple stenosis influence. Many researcher have developed various type of Mathematical models to study the flow of Blood through channel with variable cross-section (Shukla [14]) , by treating blood as a Newtonian fluid. It is seen that blood behaves Herschel-Bulkley fluid in small diameter tube (Chaturani *et.al.*[4]). Many researchers have studied the blood flow through stenosed arteries by treating as Herschel-Bulkley fluid (Biswas and Laskar [3]).

Tandon, *et. al.* [18] described a mathematical model to understand the post-stenotic dilatation problem by treating blood as Casson fluid . A K Singh *et.al* [16] have studied the resistance to the flow ratio for yield stress by considering blood flow in small artery having stenosis and post stenotic dilatation. Sanjeev *et. al.* [13] represented blood flow resistance for a small artery with multiple stenoses & post-stenotic dilatation and Maruthi Prasad *et. al.* [6] examined post-stenotic dilatation by considering the blood as a Jeffrey fluid. Pincombe *et. al.* [11] have studied influence of multiple stenosis and post stenotic dilatation in small arteries by considering blood as non-Newtonian.

Arun Kumar [1] analysed the influence of multiple stenoses and post-dilatation on blood flow through an artery by considering fluid as Herschel-Bulkley. Multiple stenotic effects on blood flow in the presence of slip was investigated by Arun Kumar [2].The flow of blood through uniform and stenosed tubes under the effect of slip velocity are studied by Philip and Chandra [10]. The effect of slip condition on fluid flow through the porous medium with stenosis by considering fluid as couple stress studied by G Radhakrishnamacharya *et. al.* [5]. By taking narrow tubes G. Radhakrishnamacharya *et. al.* [9] examined the slip effect on H-B fluid.

Priyadarshini *et. al.* [12] have analyzed the flow of Herschel-Bulkley fluid through a tapered arterial stenosis with dilatation. K M Prasad *et. al.* [7] investigated the effect of stenosis and post-stenotic dilatation by treating blood as Herschel-Bulkley fluid.

Motivated by these studies an attempt is made in this paper to study the slip effect on Stenosis and post-stenotic dilatation by considering blood as Herschel-Bulkley.

2. MATHEMATICAL FORMULATION

In the present study we considered the steady flow of H-B fluid through a circular artery containing multiple abnormal segments.

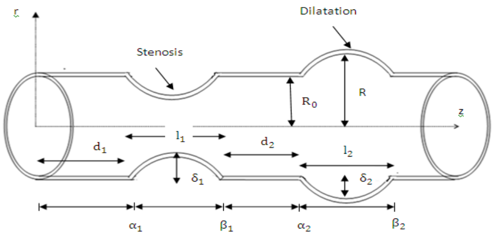


Figure. 1:Geometry of the problem

The geometry of the wall of the tube is taken as ([K Maruti Prasad *et.al* [7])

$$h=\frac{R(z)}{R_o}\begin{cases}1-\dfrac{\delta_i}{2R_o}\left[1+\cos\dfrac{2\pi}{L_i}\left(z-\alpha_i-\dfrac{L_i}{2}\right)\right], & \alpha_i\le z\le\beta_i\\ 1, & otherwise\end{cases} \tag{1}$$

The radius of the artery at dilatation , normal artery and length of the i^{th} abnormal segement is represented by R, R_o, L_i respectively. The max. distance of the i^{th} abnormal segment is given by δ_i and it is +ve , –ve for stenosis and aneurysms respectively . The distance from the origin to the beginning of the i^{th} abnormal segment is denoted by α_i and it is given as

$$\alpha_i=\left[\sum_{j=1}^{i}\left(d_j+L_j\right)\right]-L_i \tag{2}$$

The distance from the origin to the end of the i^{th} abnormal segment is given by β_i , i.e

$$\beta_i=\left[\sum_{j=1}^{i}\left(d_j+l_j\right)\right] \tag{3}$$

Where d_i is the distance of i^{th} and $(i-1)^{th}$ start and end abnormal segment respectively given by [14].

By considering the circular cross-section on an artery under mild stenosis condition the general equation is given as

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau)=-\frac{\partial p}{\partial r} \tag{4}$$

Where τ is the shear stress for H-B fluid and given by

$$\tau=\left(-\frac{\partial w}{\partial r}\right)^n+\tau_o \text{ if } \tau\ge\tau_o \tag{5}$$

$$\frac{\partial w}{\partial r}=0 \text{ if } \tau<\tau_o \tag{6}$$

Where (r,z) are cylindrical polar co-ordinates with z is along the tube where as r is along the normal to the axis of the tube. P is pressure, τ is shear stress and τ_o is yield and w is the velocity of the fluid and n is the power law index.

The boundary conditions are:

(i) $\left.\tau\right|_{r=0}$ is finite at (7)

(ii) $\left.\dfrac{\partial w}{\partial r}\right|_{r=h(z)}=\dfrac{-\alpha}{Ro\sqrt{Da}}w$ (8)

Where the slip parameter is represented by α , permeability parameter Da and radius of the tube is Ro .

3. SOLUTION

The velocity is calculated by solving equations. (5-6), by using the boundary conditions (7-8) we obtained,

$$w=\frac{h^{m+1}P^m}{2^m(m+1)}\left[\begin{array}{c}\left(1-\dfrac{2\tau_o}{hP}\right)^{m+1}-\left(\dfrac{r}{h}-\dfrac{2\tau_o}{hP}\right)^{m+1}\\ +\dfrac{R_o\sqrt{Da}}{\alpha}\dfrac{(m+1)}{h}\left(1-\dfrac{2\tau_o}{hP}\right)^m\end{array}\right] \tag{9}$$

Where $P=-\frac{\partial p}{\partial z}$ and $w=\frac{1}{n}$

The plug flow region is obtained by applying boundary condition (6)

$$r_o=\frac{2\tau_o}{P} \tag{10}$$

By applying the condition $\tau=\tau_h$ at $r=h$ we get

$$\frac{r_o}{h}=\frac{\tau_o}{\tau_h}=\tau,\ 0<\tau<1 \tag{11}$$

Taking $r=r_o$ in equation (9), the plug core velocity is

$$w_p=\frac{h^{m+1}P^m}{2^m(m+1)}\left[\left(1-\frac{r_o}{h}\right)^{m+1}+\frac{R_o\sqrt{Da}}{\alpha}\frac{(m+1)}{h}\left(1-\frac{r_o}{h}\right)^m\right]$$

$$\text{for } 0\le r\le r_o \tag{12}$$

Q is volumetric flow rate defined as

$$Q=2\left[\int_0^{r_o}w_p\,r\,dr+\int_{r_o}^{r_h}w\,r\,dr\right] \tag{13}$$

On integrating we get

$$Q = \frac{P^m h^{m+3}\left(1-\frac{r_o}{h}\right)^{m+1}}{2^m m_1} \times \left[\begin{array}{l} m_2 - 2(m+3)\left(1-\frac{r_o}{h}\right) \\ +2\left(1-\frac{r_o}{h}\right)^2 + m_1 \\ \times \frac{R_o\sqrt{Da}}{\alpha h^3}\left(1-\frac{r_o}{h}\right)^{-1}\left\{r_o^2 + h\left(1-\frac{r_o^2}{h^2}\right)\right\} \end{array}\right] \tag{14}$$

From equation (14)

$$\frac{dp}{dz} = -P = \frac{\left[2^m m_1 Q\right]^{\frac{1}{m}}}{h^{1+\frac{3}{m}}(1-\tau)^{1+\frac{1}{m}}\left[m_2 - 2m_3 + m_1\frac{m_4}{(1-\tau)}\right]} \tag{15}$$

Where

$$m_1 = (m+1)(m+2)(m+3)$$
$$m_2 = (m+2)(m+3)$$
$$m_3 = (m+3)(1-\tau) + 2(1-\tau)^2$$
$$m_4 = \frac{R_o\sqrt{Da}}{\alpha h^3}\left\{\frac{\tau^2}{h^2} + \frac{1}{h^2}\left(1-\tau^2\right)\right\}$$

The formula for calculating pressure drop Δp is

$$\Delta p = \int_0^l \frac{dp}{dz}dz \tag{16}$$

$$\Delta p = \int_0^l \frac{\left[2^m m_1 Q\right]^{\frac{1}{m}}}{h^{1+\frac{3}{m}}\left[\begin{array}{l} m_2(1-\tau)^{m+1} - 2(m+2+\tau)(1-\tau)^{m+2} \\ +m_1 m_4 (1-\tau)^m \end{array}\right]^{\frac{1}{m}}} dz \tag{17}$$

Using the following non-dimensional quantities

$$z^* = \frac{z}{L},\ \delta^* = \frac{\delta}{R_o},\ R^* = \frac{R(z)}{R_o},\ P^* = \frac{P}{\left(\mu WL/R_o\right)},$$

$$\tau_o^* = \frac{\tau_o}{\left(\mu W/R_o\right)},\ \tau^* = \frac{\tau}{\mu\left(W/R_o\right)},\ Q^* = \frac{Q}{\pi R_o^2 W}$$

In Eqn.(17) , we finally get (after dropping asterick)

$$\Delta p = \int_0^l \frac{\left[2^m m_1 Q\right]^{\frac{1}{m}}}{h^{1+\frac{3}{m}}\left[\begin{array}{l} m_2(1-\tau)^{m+1} - 2(m+2+\tau)(1-\tau)^{m+2} \\ +m_1 m_4 (1-\tau)^m \end{array}\right]^{\frac{1}{m}}} dz \tag{18}$$

λ is the flow to the resistance given by

$$\lambda = \frac{\Delta p}{Q} = \frac{1}{Q}\int_0^l \frac{\left[2^m m_1 Q\right]^{\frac{1}{m}}}{h^{1+\frac{3}{m}}\left[\begin{array}{l} m_2(1-\tau)^{m+1} - 2(m+2+\tau)(1-\tau)^{m+2} \\ +m_1 m_4 (1-\tau)^m \end{array}\right]^{\frac{1}{m}}} dz \tag{19}$$

With out stenosis ($h=1$) the pressure drop Δp_N is calculated from Eq. (18) as

$$\Delta p_N = \int_0^l \frac{\left[2^m m_1 Q\right]^{\frac{1}{m}}}{\left[\begin{array}{l} m_2(1-\tau)^{m+1} - 2(m+2+\tau)(1-\tau)^{m+2} \\ +m_1\frac{R_o\sqrt{Da}(1-\tau)^m}{\alpha} \end{array}\right]^{\frac{1}{m}}} dz \tag{20}$$

The resistance to the flow in the absence of stenosis λ_N is obtained from Eq.(20) as,

$$\lambda_N = \frac{\Delta p_N}{Q} \tag{21}$$

$\bar{\lambda}$ is the normalized resistance to the flow and it is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} \tag{22}$$

The wall shear stress is given by

$$\tau_h = -\frac{h}{2}\frac{dp}{dz} \tag{23}$$

4. RESULT AND DISCUSSISON

The equations for velocity (w) , core velocity (w_p), volumetric flow rate (Q), resistance to the flow (λ) , pressure drop (Δp)and wall shear stress (τ_h) are given and have been evaluated for various values of corresponding parameters and presented graphically.

It is illustrated from fig. 2 that as permeability parameter (Da) increases flow resistance decreases. Further increases in slip parameter (α) and pow law index increases flow resistance is also increases [figs. (3,4)].

The wall shear stress(τ_h) decreases as permeability parameter (Da) decreases and increases with slip parameter (α) [figs. (5,6)]. It is also observed that pressure drop decreases as permeability parameter (Da) increases, [fig (7)]. Figure 8 and figure 9 demonstrated the steam line in the stenotic regions. It is observed that stenosis increases the resistance to the flow increase but reverse phenomena is observed in dilation case.

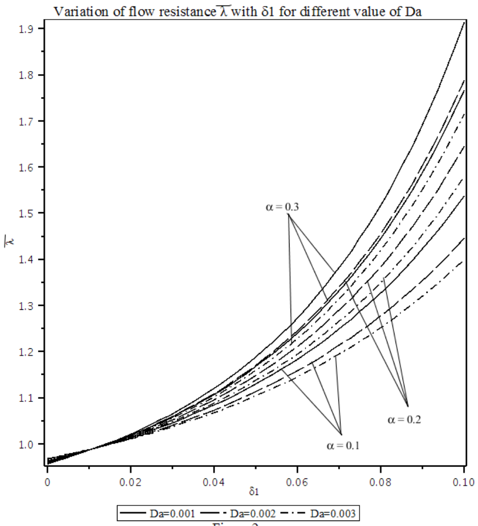
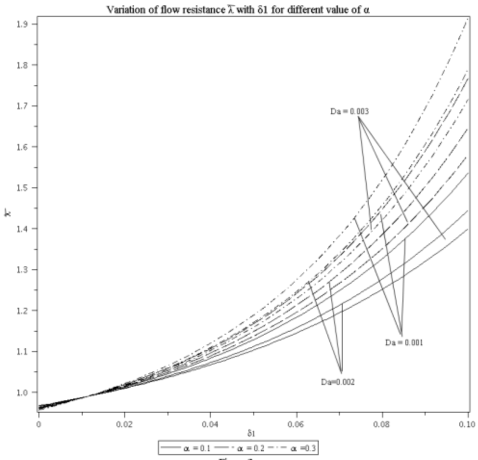


Figure.2



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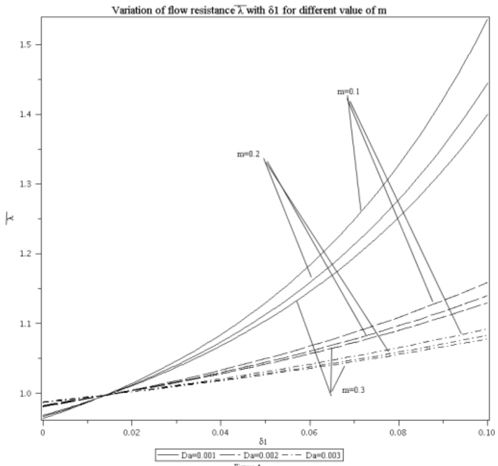

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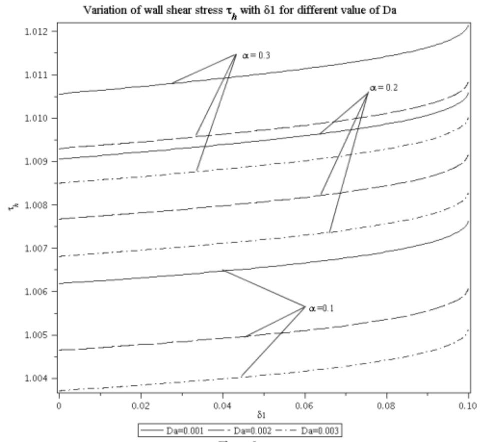

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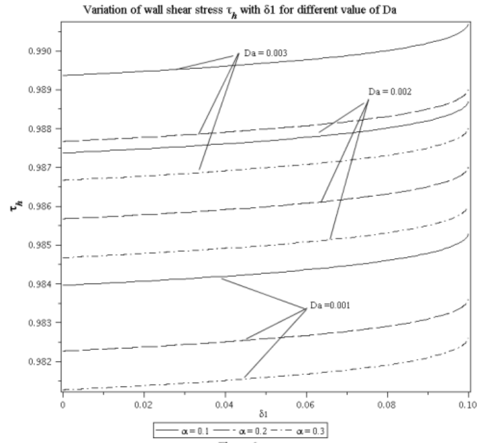

Figure 6

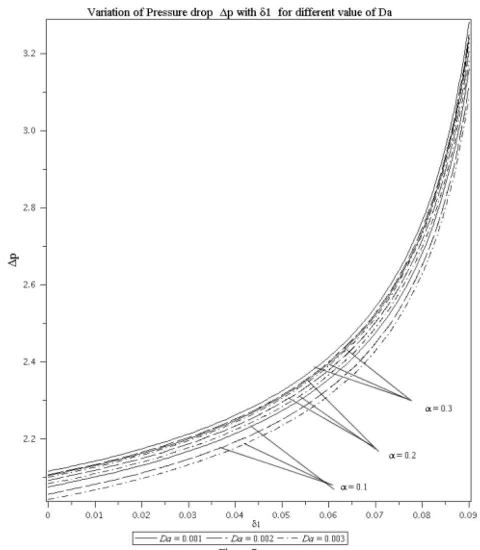

Figure 7

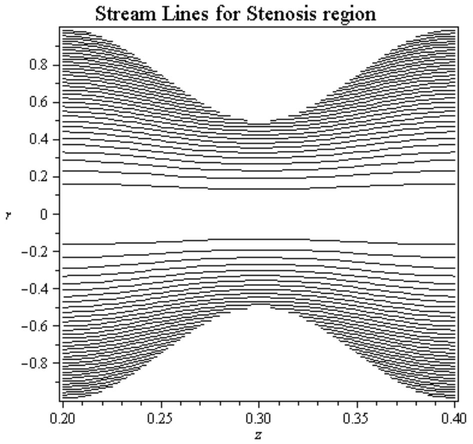

Figure 8

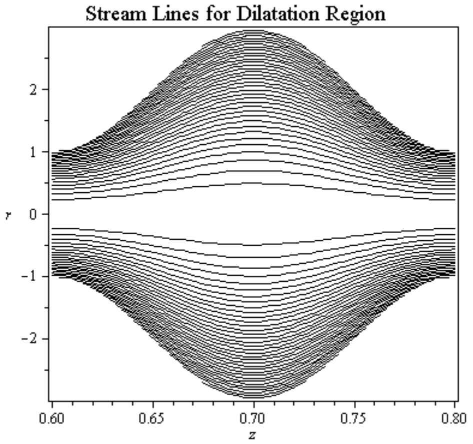

Figure 9

5. CONCLUSIONS

The steady flow of Herschel Bulkley fluid flow through an artery with both stenosis and dilatations under the influence of slip have been presented. The results have been evaluated and observed that the resistance to the flow decreases with increase with the permeability parameter (Da), power law index, and increases with slip parameter (α) . As wall shear stress decreases as the permeability parameter (Da) decreases and increases with slip parameter (α).

REFERENCES

[1] Arun Kumar Maiti.(2016): Effect of multiple stenoses and post dilatation on blood flow through an artery.International Journal of Multidisciplinary Research and Development, 3(8), pp. 362-366.

[2] Arun Kumar Maiti.(2016): Multiple Stenotic Effect on Blood Flow Characteristics in Presence of Slip Velocity. American Journal of Applied Mathematics and Statistics, **4**(6) pp. 194-198.

[3] Biswas, D.; Laskar, R. B. (2011): Steady flow of blood through a stenosed artery—a non-Newtonian fluid model. Assam University Journal of Science and Technology, **7**, pp. 144–153.

[4] Chaturani, P.; Ponnalagarsamy, R.A. (1985): Study of non-Newtonian aspects of blood flow through stenosed arteries and its applications in arterial diseases. Biorheol, **22**, pp.521-531.

[5] Gurju Awgichew, Radhakrishnamacharya, G.(2013): Effect of slip condition on couple stress fluid flow through porous medium with stenosis. International Journal of Scientific & Engineering Research, **4**(9),pp.28-30.

[6] Maruti Prasad,K.; Bhuvana Vijaya, R,; and Umadevi, C.(2015): Effects Of Stenosis and Post Stenotic Dilatations on Jeffrey Fluid Flow in Arteries. International Journal of Research in Engineering and Technology. **4**, pp.195-201.

[7] Maruti Prasad,K.; Bhuvana Vijaya, R,; and Umadevi, C.(2017): Flow of Herschel-Bulkley Fluid flow through an artery with the effect of stenosis and post stenotic dilatiation. IJMS.**8**(6), pp.189-196.

[8] Mishra, B.K.; Verma, N. (2010): Effect of stenosis on non-Newtonian flow of blood in blood vessels. Australian Journal of Basic and Applied sciences. **4**, pp.588-601.

[9] Nallapu Santhosh , Radhakrishnamacharya , G., Chamkha, Ali J.(2015): Effect of slip on Herschel–Bulkley fluid flow through narrow tubes. Alexandria Engineering Journal. **54**, pp.889–896.

[10] Philip, D.; Chandra, P.(1996): Flow of Eringen fluid (simple micro fluid) through an artery with mild stenosis. Int. J. Engng Sci. P. pp.87–99.

[11] Pincombe, B. and Mazumdar, J.(1997): The effects of Post-stenotic Dilatations on the Flow of a Blood through stenosed Coronary Arteries. Mathematical and Computer Modelling.**25**, pp.57-70.

[12] Priyadharshini, S., Ponalagusamy, R.(2015): Biorheological Model on Flow of Herschel-Bulkley Fluid through a Tapered Arterial Stenosis with Dilatation. Bionics and Biomechanics, pp.1-12.

[13] Sanjeev ,K, and Chandrashekhar, D.(2013): Blood flow Resistance for a small artery with the effect of Multiple stenosis and post stenotic dilatation. International journal of Engineering Sciences and Emerging Technologies, **6**, pp.57-64.

[14] Shukla, J.B; Parihar, R.S.; Gupta, S.(1980): Effects of peripheral layer viscosity on blood flow through the artery with mild stenosis.

Bulletin of Mathematical Biology, **42**(6), pp. 797-805.

[15] Shukla, J.B .; Parihar, R.S.; Rao, B.R.P. (1980): Effects of stenosis on non-Newtonian flow through an artery with mild stenosis. Bull.Math.Biol..**42**, pp. 283-294.

[16] Singh, A.K, and Singh, D.P., (2012): A Computational study of Bingham plastic flow of Blood through an artery by multiple stenoses and post dilatation. Advances in Applied Science Research ,**3**(5), pp.3285-3290.

[17] Sreenadh, S.; Pallavi, A.R.; Satyanarayana, B.H.(2011). Flow of a casson fluid Through an Inclined tube of Non-uniform cross-section with multiple stenoses. Adv. Appl. Sci. Res. **2**(5), pp. 340-349.

[18] Tandon, P.N.; Rana, U.V. ; Kawahara, M.; Katiyar, V.K.(1993): A model for blood flow through stenotic tube", Int. J. Biomed. Comput, **32**, pp.62-78.